

# Spectral flow in superconducting point contacts

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We find that multiple Andreev reflections mediating the transport in superconducting point contacts are strongly affected by a small amount of impurities in the area of the contact. We also argue that the model based on Zener transitions within independent conducting channels is not suitable for kinetic processes in multichannel contacts.

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## INTRODUCTION

Effect of impurities on transport in superconducting nanostructures is one of the key issues in the physics of mesoscopic superconductors, among which point contacts are the simplest devices of interest. One can distinguish contacts with only a few transverse eigenmodes (or conducting channels) relevant for a given temperature (quantum contacts) and those where the number of channels  $N_{\text{ch}}$  is relatively large (mesoscopic contacts). In quantum contacts, the effect of impurities on both static and transport properties can be satisfactorily described by introducing certain transmission and reflection probability  $D+R=1$  for each conducting channel [1]. Mesoscopic weak links with disorder are sometimes also treated within this scheme. To this end, a quantum eigenstate problem for a charge carrier is to be solved for the particular realization of disorder, each state being considered as an individual channel with certain transmission probability. The problem of a multichannel weak link is then mapped into that for a set of tunnel contacts having an effective distribution of transmission probabilities. This scheme has been shown recently [2] to be equivalent to the semi-classical theory of superconductivity for calculating some *static* properties of multichannel contacts.

The individual channels found by solving the quantum-mechanical problem are independent in the sense that transitions between them are absent under static conditions. One might now raise a question how useful is the model of independent channels for *transport* properties of multichannel contacts when time-dependent perturbations induce transitions between the individual channels. This is of a crucial importance for further theoretical studies of transport in superconducting mesoscopic multichannel devices in presence of disorder. We address this problem using a specific example of a ballistic point contact between two clean superconductors assuming that the thickness  $d$  of the insulating layer and the size  $a$  of the orifice in it are much larger than atomic dimensions but shorter than the coherence length  $\xi$  and the mean free path  $\ell$ , i.e.,  $a \sim d \ll \xi \ll \ell$ . The contact thus has a very large number of conducting channels,  $N_{\text{ch}} \sim (p_F a)^2 \gg 1$ . The spectrum of the midgap states localized within the coherence length  $\xi$  near the point contact was first found in [3]

$$\epsilon_{\pm} = \mp |\Delta| \cos(\phi/2) \quad (1)$$

for a phase difference  $\phi$  between the two superconductors. The spectrum is degenerate: all particles flying from one superconductor into another have the same energy independently of their momentum directions within the corresponding hemispheres. Another example is a contact having a tunnel barrier with a transmission probability  $D$ . The spectrum [4]

$$\epsilon = \pm |\Delta| \sqrt{1 - D \sin^2(\phi/2)} \quad (2)$$

has a (mini)gap  $\sqrt{R}|\Delta|$  near the phase difference  $\phi = \pi$ , which separates the two branches of the spectrum. The mid-gap states have a profound effect on dynamics of the contacts. When the applied voltage is larger than the inelastic relaxation rate,  $eV \gg \tau_e^{-1}$ , a nonlinear regime sets in which originates from multiple Andreev reflections (MAR) [5, 6, 7, 8, 9, 10] of localized quasiparticles. For a contact without both impurities and a barrier, excitations from below the gap edge with  $\epsilon = -|\Delta|$  are accelerated through the midgap states Eq. (1) and re-appear above the gap with  $\epsilon = |\Delta|$  and vice versa, thus producing a highly nonequilibrium situation. On the contrary, if a contact has a barrier with a reflectivity  $R$ , the localized excitations cannot accelerate due to the minigap and retain their original states. It is only Zener tunnelling through the minigap that provides nonequilibrium current through the contact [8, 9, 10].

The model of independent channels was applied to kinetic effects in multichannel point contacts in Ref. [9]. The key ingredients of that model were (i) the MAR processes in separate channels with (ii) Zener transitions through a minigap between two spectral branches in Eq. (2) within each individual channel, and (iii) a standard distribution [11] of reflection coefficients over the channels. Here we study the transport in mesoscopic multichannel contacts with impurities and show that the above model of individual channels consisting of processes (i)–(iii) is not suitable for dynamic phenomena when the number of channels is large,  $N_{\text{ch}} \gg 1$ , because transitions between different channels are excited in time-dependent conditions. As a result, the system finds the optimal way to relax through channels with small reflection probabilities avoiding weak Zener tunneling. We further demonstrate that the concept of MAR is equivalent to the *spectral flow* appearing in a wide variety of quantum systems [12, 13, 14]. Using the quasiclassical theory of nonstationary superconductivity we show that the impurity scattering provides an effective momentum exchange between particles moving from the opposite sides of the contact and impedes the MAR acceleration. For low inelastic relaxation rate, the I–V curve displays a non-trivial behavior: At low voltages, impurity relaxation slows the acceleration down and results in a new linear current-voltage dependence with a large conductance. This differs from a square-root dependence obtained within the independent-channel model (i)–(iii).

### KINETIC EQUATION

The density of states (DOS) for the midgap states in a classical point contact was calculated in Refs. [2, 3]. It is  $\nu(s) = \nu(0)e^{-k|s|}$  with  $k = 2v_F^{-1}\sqrt{|\Delta|^2 - \epsilon^2}$  where  $s$  is the distance along the particle trajectory;  $\nu$  is normalized to the normal single-spin DOS at the Fermi level,  $N_F$ . We use the standard kinetic equation approach [15, 16] where the distribution function is split into two components  $f^{(1)}$  and  $f^{(2)}$  that are respectively odd and even in the variables  $\epsilon$ ,  $\mathbf{p}$ . They are coupled by the Boltzmann equation

$$\nu v_F \frac{1}{2} \frac{\partial \dot{\chi}}{\partial s} \frac{\partial f^{(1)}}{\partial \epsilon} + \nu \frac{\partial f^{(1)}}{\partial t} + v_F \frac{\partial}{\partial s} \left( \nu f^{(2)} \right) = J. \quad (3)$$

The second equation ensures that the distribution function  $f^{(1)}$  is constant along the trajectory [16]. In Eq. (3), the first term in the l.h.s. originates from the Doppler energy and  $\dot{\chi}$  is the time derivative of the phase, and  $J$  is the impurity collision integral. A detailed discussion of the kinetic equations is given in Refs. [15, 16]. The standard technique of averaging over impurities applies since the number of impurities within the volume of the orifice is large. Indeed, the mean free time is  $\tau^{-1} \sim N_F n_{\text{imp}} |u|^2$  where  $|u| \sim U p_F^{-3}$  is the Fourier transform of the impurity potential  $U$ . The number of impurities  $n_{\text{imp}} da^2 \sim (d/\ell)(E_F/U)^2 N_{\text{ch}}$  can be very large even for  $U \sim E_F$  because of a macroscopic number of quantum channels in the orifice  $N_{\text{ch}} \gg 1$ . The large number of impurities is the essential parameter in our theory.

We integrate Eq. (3) along the trajectory across the contact over distances longer than  $\xi$ . The first term in the l.h.s. of Eq. (3) has a sharp maximum at  $s \sim d$  where  $\nu(s)$  is nearly constant. The third term in the l.h.s. disappears due to the decay of  $\nu$ . The collision integral contains [2] a combination of angular averages  $\nu [\langle \nu \rangle f^{(1)} - \langle \nu f^{(1)} \rangle]$ . We write  $\langle \nu \rangle = [\langle \nu \rangle_+ + \langle \nu \rangle_-] / 2$  where  $\langle \nu \rangle_{\pm} = (2\pi)^{-1} \int_{\pm p_x > 0} \nu(\mathbf{p}, \mathbf{r}) d\Omega_{\mathbf{p}}$ . The upper (lower) sign is for right-moving (left-moving) particles with  $p_x > 0$  ( $p_x < 0$ ). We choose the direction of the  $x$  axis from the region with the phase  $\chi_1$  into that with  $\chi_2$ . Since  $f^{(1)}$  depends on the sign of  $p_x$  only, being independent of the direction of  $\mathbf{p}$  within each hemisphere  $p_x > 0$  or  $p_x < 0$ ,

$$J_{\pm} = -\tau^{-1} \nu_{\pm} \langle \nu \rangle_{\mp} \left( f_{\pm}^{(1)} - f_{\mp}^{(1)} \right). \quad (4)$$

The collision integral Eq. (4) is only nonvanishing if  $\nu_+$  and  $\nu_-$  are nonzero simultaneously. It is localized at distances  $s \sim d$ : The averages  $\langle \nu \rangle$  decay at  $s \gtrsim a$  away from the contact, being proportional to the solid angle at which the orifice is visible from the position point. Since  $\nu$  is constant within the orifice,  $\ell^{-1} \int ds \langle \nu \rangle_{\mp} = \gamma \nu_{\mp}(0)$  where  $\gamma \sim d/\ell$  is a contact-dependent geometric factor; we assume a constant  $\gamma$  for simplicity. Finally, putting  $\phi = \chi_2 - \chi_1$  we find

$$\nu_{\pm}(0) \left[ \pm \frac{1}{2} \frac{\partial \phi}{\partial t} \frac{\partial f_{\pm}^{(1)}}{\partial \epsilon} + \frac{1}{\sqrt{|\Delta|^2 - \epsilon^2}} \frac{\partial f_{\pm}^{(1)}}{\partial t} \right] = -\gamma \nu_{\pm}(0) \nu_{\mp}(0) \left( f_{\pm}^{(1)} - f_{\mp}^{(1)} \right). \quad (5)$$

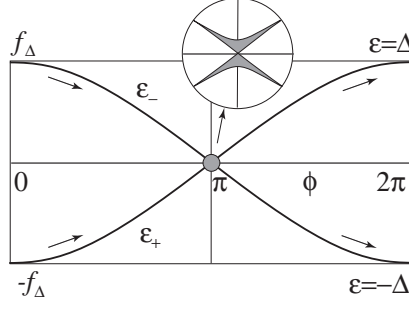


FIG. 1: Spectral flow through the midgap states. For  $\dot{\phi} > 0$ , a particle with  $p_x > 0$  is captured at  $\phi = 0$  with an energy  $\epsilon_+ = -|\Delta|$  and then released at  $\phi = 2\pi$  with  $\epsilon_+ = |\Delta|$ ; a particle having  $p_x < 0$  is captured with  $\epsilon_- = |\Delta|$  and released with  $\epsilon_- = -|\Delta|$ . The dark dot (expanded in the insert) shows the impurity band  $\sim |\Delta|\sqrt{\gamma}$  near the crossing point.

### SPECTRAL FLOW IN A BALLISTIC CONTACT

In the collisionless limit [3],

$$\nu_{\pm}(0) = \pi\sqrt{|\Delta|^2 - \epsilon^2}\delta(\epsilon - \epsilon_{\pm}). \quad (6)$$

Since  $\partial\epsilon_{\pm}/\partial\phi = \pm\frac{1}{2}|\Delta|\sin(\phi/2)$  we can write Eq. (5) as  $\nu_{\pm}(0)df^{(1)}/dt = 0$  where the total time derivative of the distribution function is taken along the spectrum Eq. (1). The total time derivative vanishes: the distribution is constant in time and thus it is independent of energy. To find it we note that, as  $\phi$  varies from 0 to  $2\pi$ , the distribution of particles with  $p_x > 0$  remains the same as it was at  $t = 0$ ,  $\phi = 0$  for an energy  $\epsilon = -|\Delta|$ , i.e.,  $f^{(1)} = -f_{\Delta}$  where  $f_{\Delta} = \tanh(|\Delta|/2T)$ . For particles with  $p_x < 0$ , the distribution remains as it was at  $t = 0$ ,  $\phi = 0$ , and  $\epsilon = |\Delta|$ , i.e.,  $f^{(1)} = f_{\Delta}$ . If  $\phi$  decreases from  $2\pi$  to 0, the signs are opposite,

$$f_{\pm}^{(1)} = \mp f_{\Delta} \text{sign}(\dot{\phi}). \quad (7)$$

Excitations are accelerated by the applied voltage undergoing multiple Andreev reflections [5, 6, 8] before their energies grow by  $2|\Delta|$  to let them escape from the potential well.

The same process can also be viewed differently. Under a small voltage bias  $eV \ll |\Delta|$ , quasiparticles are accelerated along the spectrum Eq. (1) with  $\phi = 2eVt$ . As the phase varies from 0 to  $2\pi$ , the quasiparticle energy changes from  $\mp|\Delta|$  to  $\pm|\Delta|$ . A quasiparticle that has been captured below the gap edge emerges above the gap after one period of Josephson oscillations (see Fig. 1). Another particle gets captured and then released during the next period, and so on [7]. This is known as the *spectral flow*. It plays a crucial part in vortex dynamics [13, 14]. In a context of weak links, it was first discussed in Ref. [17]. In the absence of collisions, the distribution of excitations localized near the contact Eq. (7) is determined by distribution of excitations at the gap edges, which are delocalized and are thus nearly in equilibrium due to the relaxation in the bulk. However, emerging quasiparticles have a “wrong” distribution with respect to the “native” ones; they relax producing dissipation.

The current through the contact with the Sharvin conductance  $R_N^{-1} = e^2 N_F v_F S/2$  is

$$I = -\frac{1}{eR_N} \int \frac{d\epsilon}{2} \left[ \nu_+ f_+^{(1)} - \nu_- f_-^{(1)} \right]. \quad (8)$$

where  $S \sim a^2$  is the area of the contact. With Eq. (7) the current due to Andreev states is

$$|e|R_N I = \pi|\Delta| |\sin(\phi/2)| f_{\Delta} \text{sign}(V). \quad (9)$$

Excitations with energies  $|\epsilon| > |\Delta|$  give a much smaller current, which can be neglected. Equation (9) agrees with the results of Refs. [5, 6, 8].

### SPECTRAL FLOW AND IMPURITIES

A small amount of impurities in the area of the contact lifts the degeneracy of Eq. (1) and modifies the spectrum in a way that an impurity band appears near the point  $\phi = \pi$  where the two spectral branches cross [2]. In the presence

of impurities the DOS is  $\nu_{\epsilon\pm}(0) = (E_{\mp}/2\gamma) \sqrt{(4\gamma/E_+E_-) - 1}$  where  $E_{\pm} = \epsilon/|\Delta| \pm \cos(\phi/2)$ . Both  $\nu_+$  and  $\nu_-$  are nonzero near the lines  $E_{\pm} = 0$  within the impurity band  $0 < E_+E_- < 4\gamma$  shown in Fig. 1. To simplify the partial differential equation (5) we note that, in the region far from the crossing point,  $|\epsilon| \gg |\Delta|\sqrt{\gamma}$ ,  $|\phi - \pi| \gg \sqrt{\gamma}$ , the ratio  $\nu_-/\nu_+ = E_+/E_-$  is of the order of  $\gamma|\Delta|^2/\epsilon^2 \ll 1$  near the line  $E_+ = 0$ , and it is  $\nu_-/\nu_+ \sim \epsilon^2/\gamma|\Delta|^2 \gg 1$  near  $E_- = 0$ . Therefore, far from the crossing point,  $|\phi - \pi| \gtrsim \beta$  where  $\sqrt{\gamma} \ll \beta \ll \pi$ , the DOS has the form of Eq. (6), and the l.h.s. of Eq. (5) reduces to the full derivative  $df_{\pm}^{(1)}/dt$  along the respective lines  $E_{\pm} = 0$ . Since the collision integral also vanishes for  $|\phi - \pi| \gtrsim \beta$ , the distribution is constant in time far from the crossing point, in the same way as it was without impurities.

Equation (5) can be solved by perturbations in the limit of small voltages. The zero-order term corresponds to vanishing of the l.h.s. and results in the condition  $f_+^{(1)} = f_-^{(1)}$  that holds through the crossing point for a given energy. We find that the distribution functions at the four lines in Fig. 1 entering the crossing region satisfy  $f_{\pm}^{(1)}(\pi + \beta) = f_{\mp}^{(1)}(\pi - \beta) = \pm f_{\Delta}$ . To the zero approximation, particles emerging in the continuum with an energy  $\epsilon = \pm|\Delta|$  have the distribution  $\pm f_{\Delta}$  that corresponds to the local equilibrium at the gap edges

$$f_{\pm}^{(\text{eq})} = \mp \text{sign}[\cos(\phi/2)] f_{\Delta}. \quad (10)$$

In this approximation, no relaxation in the continuum occurs. The deviation  $f_{1\pm}$  from the local equilibrium is proportional to the time derivative, i.e., to the voltage. We write  $f_{\pm}^{(1)} = f_{\pm}^{(\text{eq})} + f_{1\pm}$ . The l.h.s. of Eq. (5) can be estimated as  $\nu eV (f_{\Delta}/\epsilon^*)$  where  $\epsilon^* \sim |\Delta|\sqrt{\gamma}$  is the characteristic scale of energy near the crossing point. Using  $\nu \sim 1/\sqrt{\gamma}$  we find  $f_{1\pm} \mp (eV/|\Delta|\gamma) f_{\Delta}$ . The deviation  $f_{1\pm}$  is thus linear in  $V$  for low voltages.

In general,  $f_{\pm}^{(1)}$  can be found by matching the four constants  $f_{\mp}^{(1)}(\pi \pm \beta)$  while integrating Eq. (5) across  $\phi = \pi$ . We note that for a contact with a barrier of reflectivity  $R = 1 - D$ , the excitation spectrum is given by Eq. (2) with a gap  $\sqrt{R}|\Delta|$  that separates states with positive energies from those with negative energies. As a result, the distribution for excitations with  $\epsilon > 0$  is decoupled from that for  $\epsilon < 0$  unless the Zener tunneling mixes them [8, 9]. On the contrary, in macroscopic contacts with impurities, the states near  $\phi = \pi$  form a continuum where the regions  $\epsilon > 0$  and  $\epsilon < 0$  are connected along the lines of a nonzero DOS passing through  $\phi = \pi$ , see Fig. 1. According to Eq. (5), the distribution of excitations is thus continuous within the entire region of nonzero DOS. Therefore, the distributions on all four lines entering the crossing region in Fig. 1 are coupled to each other.

Since we are interested the asymptotic values of  $f_{\pm}^{(1)}$  for  $\phi = \pi - \beta$  and  $\phi = \pi + \beta$ , we neglect the fine structure of  $\nu$  and approximate it by Eq. (6). The collision integral is then nonzero only at the crossing point of  $\epsilon_+$  and  $\epsilon_-$  defined by Eq. (1). Equation (5) yields

$$\frac{df_{\pm}^{(1)}}{dt} = \mp \pi \gamma |\Delta|^2 \Gamma \delta(\epsilon - \epsilon_{\mp}). \quad (11)$$

$\Gamma$  is a combination  $f_+^{(1)}(\epsilon, \phi) - f_-^{(1)}(\epsilon, \phi)$  taken in the vicinity of  $\epsilon = 0$ ,  $\phi = \pi$ , which couples the four values  $f_{\pm}^{(1)}(\pi \pm \beta)$ . We shall see that  $\Gamma$  varies slowly near  $\phi = \pi$  thus we approximate

$$\Gamma = \frac{1}{2} \left[ f_+^{(1)}(\pi + \beta) + f_+^{(1)}(\pi - \beta) - f_-^{(1)}(\pi + \beta) - f_-^{(1)}(\pi - \beta) \right].$$

We integrate Eq. (11) over time along the dependence  $E_+ = 0$  for particles with  $p_x > 0$  or along  $E_- = 0$  for  $p_x < 0$  and obtain

$$f_{\pm}^{(1)}(\pi + \beta) - f_{\pm}^{(1)}(\pi - \beta) = \mp \alpha \Gamma \quad (12)$$

where  $\alpha = \pi|\Delta|\gamma/2eV$ . Adding the two Eqs. (12) we find that the quantity

$$A(\beta) = f_+^{(1)}(\pi + \beta) - f_-^{(1)}(\pi - \beta) \quad (13)$$

satisfies  $A(\beta) = A(-\beta) \equiv A$ , i.e., it is a slowly varying function near  $\beta = 0$ . This proves that  $\Gamma = [A(\beta) + A(-\beta)]/2$  can indeed be treated as a constant,  $\Gamma = A$ .

Consider first the case  $\dot{\phi} = 2eV > 0$ . The initial condition at  $t = 0$ ,  $\phi = 0$  is the equilibrium distribution at an energy  $\epsilon = \mp|\Delta|$ , i.e.,  $f_{\pm}^{(1)} = \mp f_{\Delta}$ . Since the distribution is constant outside the crossing region, we have also  $f_{\pm}^{(1)}(\pi - \beta) = \mp f_{\Delta}$ . To find the functions  $f_{\pm}^{(1)}(\pi + \beta)$  we use Eqs. (12, 13) whence  $A = -2f_{\Delta}/(1 + \alpha)$  and

$$f_{\pm}^{(1)}(\pi + \beta) = \mp [(1 - \alpha)/(1 + \alpha)] f_{\Delta}.$$

For weak scattering  $\alpha \ll 1$ , the distribution remains unchanged as compared to its initial value,  $f_{\pm}^{(1)}(\pi + \beta) = f_{\pm}^{(1)}(\pi - \beta) = \mp f_{\Delta}$ . Quasiparticles relax towards the local equilibrium  $\pm f_{\Delta}$  as they emerge in the continuum from inside the gap at an energy  $\pm|\Delta|$ . If scattering is strong  $\alpha \gg 1$ , the distribution changes considerably at the crossing point  $\phi = \pi$  due to exchange between particles moving from the opposite sides of the contact. Emerging particles with an energy  $\epsilon = \pm|\Delta|$  have a distribution close to the “local equilibrium” Eq. (10) such that no relaxation occurs. This is not the true equilibrium for  $|\epsilon| < |\Delta|$ : the latter can only be established when the spectral flow rate  $eV$  is much smaller than the inelastic rate. The local equilibrium is achieved through a very effective branch mixing due to the momentum exchange on impurities when particles pass through the crossing region of the spectral branches. The deviation from the local equilibrium is zero as long as  $\phi < \pi$  while it appears for  $\phi > \pi$ :

$$f_{1\pm} \equiv f_{\pm}^{(1)}(\pi + \beta) - [\pm f_{\Delta}] = \mp [2/(1 + \alpha)] f_{\Delta}. \quad (14)$$

This agrees with our earlier perturbative estimate for the low-voltage limit.

Calculating the current for  $\phi < \pi$  from Eq. (8) we find  $I = I_s$  where the “supercurrent” is defined through the “equilibrium distribution” Eq. (10):

$$eR_N I_s = \pi |\Delta| \sin(\phi/2) f_{\Delta} \text{sign}[\cos(\phi/2)].$$

This expression differs from the well-known result for a steady-state supercurrent [3]. The reason is that Eq. (10) is not the true equilibrium, see the discussion above. For  $\phi > \pi$  we have  $I = I_s + I_n$ ; the “normal” current contains the deviation from equilibrium Eq. (14). Considering also the case  $\dot{\phi} = 2eV < 0$  we find for the normal current

$$|e|R_N I_n = 2\pi |\Delta| (1 + |\alpha|)^{-1} |\sin(\phi/2)| f_{\Delta} \text{sign}(V).$$

As in Ref. [9], it exists only when  $\pi < \phi < 2\pi$  for  $\dot{\phi} > 0$  or when  $0 < \phi < \pi$  for  $\dot{\phi} < 0$ .

For a voltage-biased contact, the supercurrent vanishes after averaging over time:

$$|e|R_N \bar{I} = 2|\Delta| (1 + |\alpha|)^{-1} f_{\Delta} \text{sign}(V). \quad (15)$$

For high voltages,  $|\alpha| \sim |\Delta|\gamma/eV \ll 1$ , the MAR result Eq. (9) is reproduced. For low voltages  $|\Delta|\gamma \gg |eV| \gg \tau_{\epsilon}^{-1}$ , the current is linear,  $\bar{I} = GV$  where  $G = (4/\pi\gamma R_N) f_{\Delta}$ . This conductance is much larger than the contribution  $\sim R_N^{-1}$  from the states with  $|\epsilon| > |\Delta|$ . To our knowledge, this low-voltage region has not been studied experimentally though the contacts with suitable parameters are now under intensive investigation [18].

## DISCUSSION

The linearity of the I-V curve for low voltages disagrees with a square-root behavior predicted in [9] using the model based on Zener transitions within independent channels averaged over the universal distribution of reflection coefficients [11] valid for diffusive conductors. This distribution cannot be applied directly for nearly ballistic contacts. The proper distribution for contacts with a small disorder was found [2] to be  $P(R) = (2\pi\gamma)^{-1} \sqrt{\gamma - R}/\sqrt{R}$ . Its denominator does also contain  $\sqrt{R}$  like the universal distribution, which, at the first glance, should have also resulted in a square-root current-voltage dependence. However, the kinetic-equation approach applicable for a multichannel contact leads to a different result. The reason is that, in a macroscopic contact, the quantum channels are very close to each other in energies with the level spacings inversely proportional to the number of channels  $N_{\text{ch}}$ . Thus, in addition to the transitions within each channel (which are the only ones considered in the independent-channel model), time dependent processes excite even more effectively transitions *between different channels*. As a result, the distribution function is spread over the entire energy interval of nonzero DOS or over the range  $R \sim \gamma$  of reflection coefficients. The role of each particular channel within the range  $\delta R$  should thus be weighted with  $\delta R/\gamma$ . If we now average the low-voltage Zener processes having the rate  $\propto \exp(-|\Delta|R/eV)$  with  $P(R)$  we would obtain the essential reflection coefficients  $R \sim eV/|\Delta|$ . Thus the fraction of essential channels contributing to the current in multichannel contacts is decreased by a factor  $eV/|\Delta|\gamma$  as compared to the independent channel model. This yields a much smaller current compared to what is predicted by the kinetic-equation approach. Therefore, the Zener processes can be neglected in favor of the momentum relaxation on impurities. To summarize, the model based on Zener transitions within independent channels does not seem to be suitable for kinetic processes in multichannel contacts. We believe, however, that calculations using the Landauer approach with the full account of interchannel transitions followed by the appropriate averaging would be equivalent to the kinetic equation technique.

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